Int. J. Heat Mass Transfer. Vol. 19, pp. 693–695. Pergamon Press 1976. Printed in Great Britain

SHORTER COMMUNICATIONS

ENTRANCE REGION HEAT TRANSFER IN FLOWING SUSPENSIONS

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(Received 7 July 1975)

NOMENCLATURE

A, area for heat transfer;
C_p, pure fluid heat capacity;
k, pure fluid thermal conductivity;
L, length of heated section;
Nu, $\frac{QR}{kA\Delta T}$, Nusselt number;
Pe, $\frac{2\rho C_p\langle v\rangle R}{k}$, Péclet number;
Q, heat-transfer rate;
r, radial coordinate;
R, tube radius;
ΔT, average temperature difference between tube wall and fluid core;
u, axial velocity;
$\langle v\rangle$, average axial velocity.

Greek symbols
δ, particle free annular layer thickness, dimensionless with respect to tube radius;
μ_a, apparent suspension viscosity (experimental);
μ_1, viscosity of pure fluid;
μ_2, suspension viscosity (theoretical);
ρ, pure fluid density;
φ, particle volume fraction.

INTRODUCTION

It is widely recognized that the addition of suspended particles serves to significantly increase rates of energy and mass transfer to a moving fluid. However, the mechanism for this important phenomena, particularly under laminar flow conditions, remains poorly understood. Both heuristic [1] and theoretical arguments [2, 3] suggest that enhanced heat-transfer rates will result from an increase in the local effective conductivity induced by the motion of suspended particles. To date, however, unambiguous experimental confirmation of this mechanism is lacking [1, 4].

An alternate mechanism, dependent on the existence of a particle-depleted layer adjacent to the solid surface, is proposed. The presence of particle-poor zones has been reported previously for a variety of suspension flow configurations [5–7]. As discussed by Ho and Leal [8] and shown experimentally by Cox and Mason [9], such non-uniform particle distributions will result in a radially varying local suspension viscosity and therefore a blunting of the velocity profile compared to that for a Newtonian fluid in laminar flow. Accordingly, in the thermal entrance region, such an increased velocity gradient at the tube wall will facilitate heat or mass transfer since these transport processes are proportional to the 1/3 power of the wall velocity gradient [10]. Based on this hypothesis, increases in entrance region heat-transfer rates would be independent of the particle physical or thermal properties and would be affected only by the thickness of the particle-poor layer near the tube wall. As will be shown, theoretically computed heat-transfer rates based on this model were found to be in substantial agreement with those measured experimentally in the entrance region.

HEAT-TRANSFER MODEL

The proposed model is based on the assumption that the flowing suspension may be described by the coaxial flow of two immiscible fluids [11]: the central core comprising a uniform suspension of solids concentration φ and viscosity μ_2, and an annular region of a particle-free fluid of viscosity μ_1. Under these circumstances, it may be shown that the actual velocity gradient at the tube wall relative to that for Poiseuille flow ($\varphi = 0$) at the same flow rate is given by:

$$\frac{\left(\left.\frac{du}{dr}\right|_{r=R}\right)_{\varphi}}{\left(\left.\frac{du}{dr}\right|_{r=R}\right)_{\varphi=0}} = \left[(1-\delta)^4\frac{\mu_1}{\mu_2}+1-(1-\delta)^4\right]^{-1} \quad (1)$$

where $\delta \cdot R$ is the thickness of the particle-free zone. It is convenient to define the Nusselt number as:

$$Nu = \frac{QR}{kA\Delta T} \quad (2)$$

where Q is the rate of heat transfer through area A, R the tube radius, k the pure fluid thermal conductivity, and ΔT the mean temperature difference between the heated tube wall and core fluid. Defining the Péclet number as

$$Pe = \frac{2\rho C_p R\langle v\rangle}{k} \quad (3)$$

where $\langle v\rangle$ is the average flow rate, and ρ and C_p are the pure fluid density and heat capacity, respectively, we can easily obtain the Lévêque [10] solution for the average rate of heat transfer to the fluid modified for the presence of the particle-depleted region,

$$Nu(\varphi) = 1.10(R/L\,Pe)^{1/3}\left[(1-\delta)^4\frac{\mu_1}{\mu_2(\varphi)}+1-(1-\delta)^4\right]^{-1/3}\left(\frac{\mu_{1b}}{\mu_{1w}}\right)^{0.14}. \quad (4)$$

Here, L is the length of the heated section and the factor $(\mu_{1b}/\mu_{1w})^{0.14}$ is an empirical correction [12] for the temperature-dependent fluid viscosity; μ_{1b} and μ_{1w} are the pure fluid viscosity at the core and wall temperatures, respectively.

The expression given by equation (4) is expected to be valid only in the thermal entrance region, for which:

$$Pe \gg 2L/R \times 10^2 \quad (5)$$

where the thermal boundary layer is of thickness $O(R \cdot Pe^{-1/3})$. For the case of coaxial flow considered herein, it must also be true that the thermal boundary layer lies entirely within the particle-free layer, hence:

$$\delta > Pe^{-1/3}. \quad (6)$$

For the experiments in this study, this requires that $Pe > O(10^4)$ since $\delta \sim 0.05$.

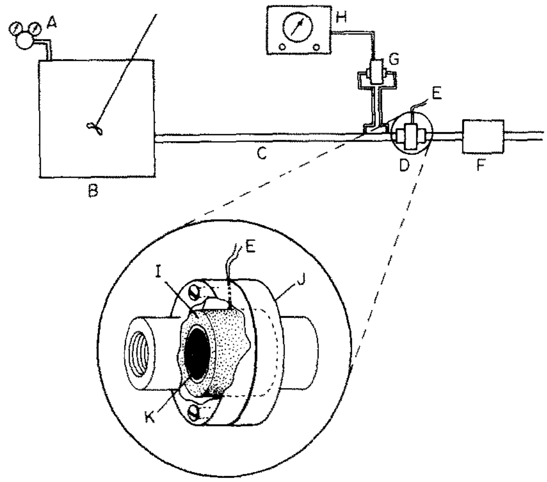

FIG. 1. Experimental apparatus: (A) Pressure regulator, 0–25 psig; (B) Pressure-tight reservoir; (C) Calming section, 75 cm transparent 1/2 in I.D. PVC tubing; (D) Heat-transfer section; (E) Power supply connections; (F) Optical distortion reducing section; (G) Pressure transducer; (H) Pressure indicator; (I) Cork-filled insulating region; (J) Lucite mount; (K) Heat-transfer measuring coil.

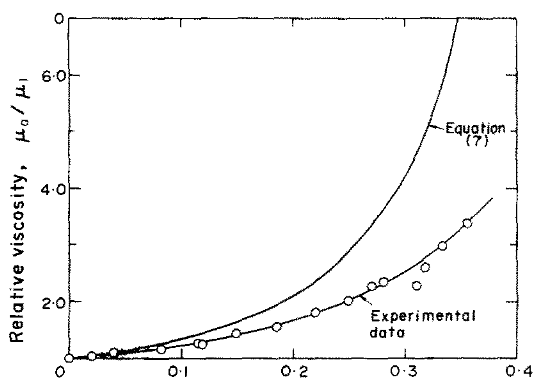

FIG. 2. Relative viscosity vs suspension concentration. ○, experimental data; —— equation (7).

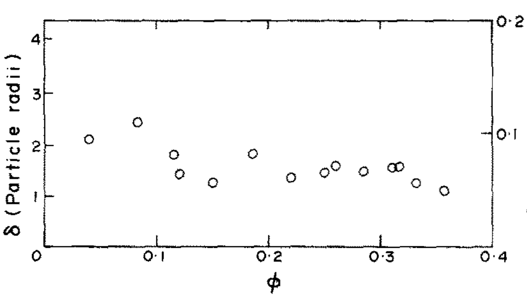

FIG. 3. Particle-free layer thickness, δ, vs concentration, computed from equation (8).

EXPERIMENTAL

With the apparatus shown in Fig. 1, rates of heat transfer were measured from a short section of electrically heated tube wall to laminar flowing suspensions of neutrally buoyant 580 μm (± 10%) dia polystyrene spheres in a mixture of Ucon polyglycol oils (36% 75-HB-450, 64% 50-HB-170). By simultaneously measuring the voltage and current in the heating coil placed in direct contact with the fluid, the rate of heat transfer to the suspension was determined at solids concentrations between 0 and 46% by volume and at flowrates between 1 and 100 cm^3/s.

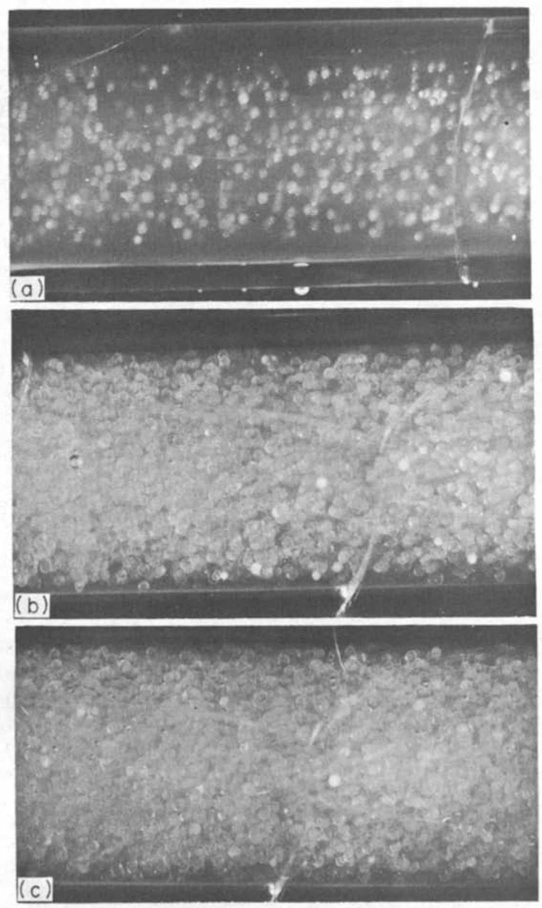

FIG. 4. Photographs of the flowing suspension: (a) $\varphi = 0.02$, $Q = 83\,\text{cm}^3/\text{s}$; (b) $\varphi = 0.11$, $Q = 19\,\text{cm}^3/\text{s}$; (c) $\varphi = 0.19$, $Q = 40\,\text{cm}^3/\text{s}$.

Estimates of the apparent suspension viscosity for various solids concentrations were obtained by measuring the pressure drop over a small section immediately upstream of the heated segment. The apparent viscosity, μ_a, relative to that of the pure fluid, μ_1, is shown in Fig. 2 as a function of particle concentration. Also shown for comparison is the relative viscosity μ_2/μ_1, where μ_2 is the viscosity predicted for a well-dispersed suspension [13]:

$$\mu_2 = \mu_1 \exp\left(\frac{2.5\varphi}{1-1.6\varphi}\right). \tag{7}$$

Based on the coaxial flow model discussed previously, the measured effective viscosity, μ_a, provides an estimate of the particle-free layer thickness as follows:

$$\delta = 1-\left(\frac{1-\mu_1/\mu_a}{1-\mu_1/\mu_2}\right)^{1/4}. \tag{8}$$

As shown in Fig. 3, these experimental estimates of δ lie in the range $0.05 < \delta < 0.1$, corresponding to one or two particle radii.

A second estimate of the particle-free layer thickness was provided by photographs of the flowing suspension taken through an optical distortion reducing section immediately downstream of the heated segment. Typical photographs, shown in Fig. 4, indicate the existence of layers near the tube wall of decreased particle concentration having thicknesses which are in good agreement with those estimated from the viscosity data (equation 8) and those previously reported by Cox and Mason [9].

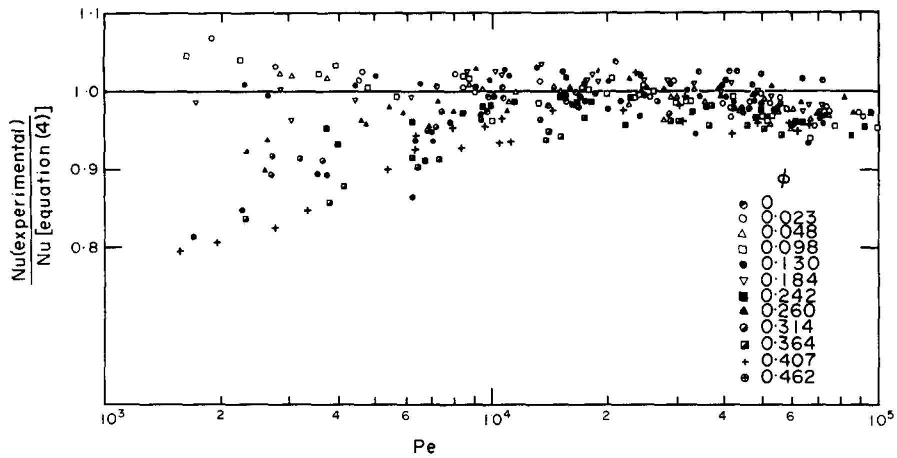

FIG. 5. *Nu* (experimental)/*Nu* (equation 4) vs Péclet number for $0.0 \leqslant \varphi \leqslant 0.462$; $\delta = 0.045$.

DISCUSSION

Experimentally measured heat-transfer rates compared with those predicted by equation (4) are shown in Fig. 5 as a function of Péclet number. The bulk suspension viscosity used in equation (4), μ_2, is calculated from equation (7). Since, as discussed previously, the parameter δ was found independently to be of the order of one to two particle radii, a value of $\delta = 0.045$, corresponding to exactly one particle radius, was used in equation (4). As expected, the sensitivity of the predicted heat-transfer rates to this parameter is greatest at the higher particle concentrations. However, a value of $\delta = 0.06$ reduces the predicted heat-transfer rate approximately 8% for $\varphi = 0.46$ and substantially less at decreased concentrations. Thus, while predicted heat-transfer rates do depend on the value chosen for δ, equation (4) remains useful so long as a physically reasonable value is chosen for δ (in this case $0.04 < \delta < 0.06$).

The excellent agreement between the experimental results and the predicted heat-transfer rates for $Pe > 10^4$, the lower limit of validity according to equation (6), demonstrates the usefulness of this model for predicting suspension heat-transfer rates. It should be re-emphasized that the predicted heat-transfer rates are independent of the particle properties and dependent only on the flow geometry (δ), velocity, and pure fluid properties.

As expected, there is less agreement between the measurements and the theoretical predictions for $Pe < 10^4$, since the thermal boundary layer penetrates the more viscous core region, thereby sharply reducing the velocity gradient in the thermal layer and thus reducing the heat-transfer rate. In contrast to the high flowrate ($Pe > 10^4$) case discussed previously, the particle properties could now become important. To confirm this, an accurate numerical solution was developed which accounts for the altered velocity profile, the bulk average thermal properties, and the temperature dependence of the fluid viscosity. These numerically predicted heat-transfer rates were found to be within 5% of those experimentally measured over the entire range of concentrations and flowrates studied.

CONCLUSIONS

The excellent agreement between experimentally measured heat-transfer rates and those predicted by equation (4) for $Pe > 10^4$ establishes the usefulness of this model for the prediction of heat-transfer rates to flowing suspensions over a wide range of concentrations and flowrates, provided the thermal boundary layer lies entirely within the particle-free zone characteristically found at the solid–suspension interface. This limits the use of equation (4) to thermal entrance regions as stipulated by the criteria of equations (5) and (6).

These results provide strong evidence that enhanced heat-transfer rates in entrance regions are primarily due to increased tube wall shear rates and secondarily to temperature dependent variations in the suspending phase viscosity. As such the thermal properties of the suspending particles themselves are of little consequence in determining heat-transfer rates in the entrance region.

Acknowledgements—This work was supported in part by a grant from the National Science Foundation, NSF-GK-36515X. The assistance of Dr. G. M. Homsy in the preparation of this manuscript is gratefully acknowledged.

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